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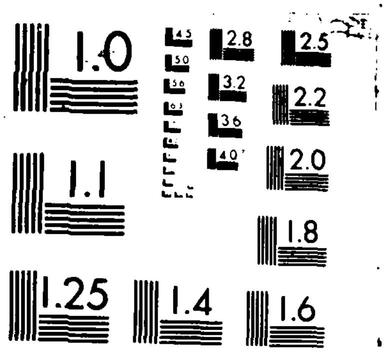

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NATIONAL BUREAU OF STANDARDS 1963-A

AFOSR-TR- 87-0214

Final Report

for

United States Air Force

Grant: AFOSR-84-0237

MultiDimensional Combustion Simulation

by

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SECURITY CLASSIFICATION OF THIS PAGE

I. Equipment Purchased.

A. List of Acquired Items:

Over the past two years, we utilized grant funds to purchase the following equipment:

Table

Vendor		Cost	Description
1) NW Marketing		$ 8,597	Mbytes (6)
2) Graham		1,962	EPOCH
3) Dynamic		5,699	Line printer
4) Datalease		42,599	Massbus disk
5) Digital		1,379	DZ-11 expansion
6) Dynamic		243	Stand
7) Talaris		14,543	Laser printer
8) Spocad		22,718	Terminals
9) Able Computer		6,373	Cluster controllers
10) Consolidated		9,751	Power conditioner
11) Digital		180	Reference manual
12) Digital		3,031	VAX/VMS Right to copy
13) Systems & Comp.		2,850	J-NET
14) Systems & Comp.		875	SLD modems
15) Telemart		578	Modems
16) Wyle Elect.		4,875	Terminals
Subtotal		126,253	
	(Minus)	1,053	(to be supplemented by WSU)
Total		125,200	

Thus, the remaining grant fund balance is zero.

B. Discussion of Variation from Final Approved List.

We recall that the modified original list as approved by AFOSR-84-0237 Amendment A on 2 February 1985 was as follows:

1)	Fixed Media Mass bus Disk and Controller	$43,300
2)	Memory Expansion	23,800
3)	Graphics Printer Plotter	10,600
4)	A synchronous Multiplexer	2,250
5)	Graphics Terminal	2,700
6)	Dual Trace Oscilloscope	1,650
7)	600 ft. ½" Tapes (20)	250
8)	1200 ft. ½" tapes (20)	400
9)	2400 ft. ½" tapes (20)	1,200
10)	Cables for Peripherals/Terminals	1,600
11)	Laser Printer	10,000
12)	Surge Protector	11,000
13)	Smoke and Temperature Alarm System	2,000
14)	Tape Drive	8,000
15)	Net Working Hardware and Software Dec Net	7,000
16)	Interface Ports	1,275
17)	Graphics Terminal	2,000
18)	Color Graphics Terminal	5,000
19)	Alpha Numeric Terminals	6,000
20)	Modems (2)	1,500
21)	Upgrade existing VT101 to do graphics	5,000
22)	Bid Cushion	277

We remark that the figures by the items represent the original estimates that we were able to obtain for the grant request. Their total exceeds the grant amount because items 11) to 22) were estimated bid savings on items 1) - 10). We list the original estimates here only for discussion purposes below.

We shall discuss each item in the above list.

Item 1. Fixed Media Massbus disk and Controller. We obtained three Fujitsu 2298 disks for the approximate price of one DEC drive.

Item 2. Memory Expansion. We obtained 6 megs of memory at a considerable savings which generated our interest in applying for and obtaining Amendment A.

Item 3. Graphics Printer Plotter. When we decided to seek Amendment A containing the laser printer, we decided that an upgrade of our line printer capability would enhance the operation of our VAX.

Item 4. Asynchronous Multiplexer: We acquired an 8 port expansion interface at considerable savings and decided to request additional ports in Amendment A.

Item 5. Graphics Terminal: We defer discussion of this until item 21.

Item 6. Dual Trace Oscilloscope: We decided that this item was not as useful as others and we decided to lump the estimated $1,650 into the terminal pool which we discuss below in item 21.

Items 7, 8 and 9. Tapes: These were purchased at the cost of $1,962 which exceeded the bid estimate of $1,850.

Item 10: Cables for Peripherals/Terminals: This item is discussed below in item 21.

Item 11. Laser Printer: We decided that this item was sufficiently significant to the presentation of results of our research effort that we exceed our $10,000 estimate by $4,543 to obtain a high quality printer and supporting software.

Item 12. Surge Protector: We saved about $1,250 on the purchase of this item.

Item 13. Smoke and Temperature Alarm System: At this writing we are engaged in further study of this item. We have not been able to locate an appropriate system at a price we can afford. In other words, we failed to estimate this item accurately enough.

Item 14. Tape Drive: Due to our acquisition of enormous disk space, we set the priority of the tape drive lower than other items. When we bid the tape drive, the bids came in @ $12,000 and the ~ $8,000 remaining in the grant at that time was insufficient to cover it. Hence, we shifted those funds to other items that are discussed below.

Item 15. Net Working Hardware and Software Dec Net: Instead of Dec Net, we purchased 2 SLD modems, Bitnet national hookup and the J-net software. Our rationale was that data transfer on a national and international scale would be a better investment than a campus network system. We discovered later that J-net as well as the interface parts discussed in the next item required the VAX-VMS 4.3 system which cost $3,200. Our justification for the acquisition of VAX-VMS 4.3 is the necessity of operating the hardware-software for Bitnet and the interfacing of numerous terminals in faculty offices to the VAX. See the discussion of the next item and item 21.

Item 16. Interface Ports: The University has promised the Mathematics Department a new location and expansion room in a renovated dormitory. The faculty offices and computer facilities will be spread over three separate floors. Thus, the additional 8 ports would not satisfy our interface needs. We decided to purchase the Mux Master interface with a 32 port capability. This system allows for a single line from the VAX to a box on each floor. The terminals on each floor plug into the box on that floor. The system can be expanded to 128 ports. However, the cost $6,373 exceeded the estimated $1,275 for 8 ports by ~ $5,100. Our justification for this expenditure is simply the necessity of interfacing the terminals in the faculty offices into the VAX. The original 8 ports purchased in Item 4 have been saturated and the number of faculty terminals spread in offices over the various floors in our new quarters would exceed the 8 new ports. This option of the Mux Master interface seemed to us to be the most efficient utilization of the funds and equipment which would allow flexibility and expansion.

Item 17, 18, and 19. Terminals: See the discussion in Item 21.

Item 20. Modems: These were purchased for $578.00.

Item 21. Upgrade VT101 to graphic terminals. We discuss at this point items 5, 10, 17, 18, 19 and 21. Lumping all of the estimates together resulted in $23,350 for terminals of various types. We decided to purchase a Lexidata 1260 color graphics terminal, 9 VT240 black and white graphics terminals, and some cables for $22,718.

Item 22. Bid Cushion: This sum has been lumped into the purchases listed above.

Summary: We believe that we have done an effective job utilizing the grant funds to achieve the optimal system for our usage and we believe that we have adhered to the spirit of the grant if not the precise detail.

II. Summaries of Past Research Work on the VAX 11-750.

A. Multidimensional Combustion Simulation.

The movement of the gas-solid interface in the combustion of a solid can be reduced to a nonlinear first order hyperbolic system of partial differential equations for which the system of parabolic reaction diffusion equations act as a nonlinear source term. Some finite difference schemes are under study for implementation. Galerkin schemes will be deferred until a later date due to the complicated structure of the system.

A Galerkin procedure for the one-dimensional problem of the combustion of a solid is currently under study and a computer program is being written for the study.

B. Other Past Research Work.

This section consists of short statements by our faculty and graduate students on their usage of the VAX.

Kurt Albrecht

Graduate Student

I now have stored on the VAX a program that would numerically approximate the solution to a free boundary problem related to the combustion of a solid (a forward difference). This program has not yet been run.

K. A. Ariyawansa

Faculty

The VAX was and is being used by myself and my students (W. S. Liang and Danny Lau) in numerical computing related to the following.

1. Optimization algorithms based on local conic approximations. These algorithms extend currently used algorithms by using local projective transformations instead of the usual local affine transformations. The additional freedom in the projective transformations can be used to make the resulting algorithms possess many desirable features. The VAX is being used to numerically test these algorithms.

2. Statistical inference for problems that possess certain structural properties.
When statistical inference problems possess certain graph structures inferential statements on the unknown parameters can be made without using external principles (like unbiasedness, minimum variance, assumption of a prior). Obtaining numerical values using this method needs computation of certain multiple integrals. The VAX was used to numerically evaluate these integrals.

3. The VAX and the plotting software and hardware we have are being used to prepare various plots necessary in the efforts 1 and 2 above.
4. Extensive use of TEX we have on our VAX is being used to prepare technical documents related to efforts 1 and 2 above.

David C. Barnes

Faculty

My last paper in SIAM J. MATH ANAL. "Extremal Problems for Eigenvalue Functionals" used the VAX to calculate and plot an extremal curve for a variational problem. The graph was done on the Soltec and is printed in the paper.

My current project is concerned with functions of eigenvalues and will do the same kind of thing for a more general problem.

I just had a paper accepted by SIAM J. Anal. "Some Approximation Formula for Stochastic Eigenvalues". This work used the VAX to calculate some examples and I was able to look at them and prove some things but the computations will not appear in print.

I have used the VAX in many other ways to calculate examples and gain insight into problems, but this work may not appear in print.

Lisa Ciasullo

Graduate Student

At present, I have about 130 files in my VAX account; most of them are subroutines used in my Math 702 project, the others are programs written for Math 544. At present, I will be using the VAX to finish up work on my Math 702 project. This will entail running subroutines, the longest of which is about 600 lines.

James A. Cochran

Faculty

Waveguide Electromagnetic Propagation

Continuing work initiated while on professional leave in Australia, additional numerical studies of microwave propagation in corrugated "cylindrical" waveguides have been carried out. In the formulation under investigation, the problem devolves to the algebraic eigenvalue problem associated with nonstandard truncations of complex-valued matrices of infinite order. Results exhibiting the expected mode mixing and/or mode suppression, as well as some unexpected eigenvalue bifurcation, have been obtained. This project continues with the support of two graduate assistants.

Bessel Function Programs

With the assistance of a Master's degree student, four FORTRAN computational routines for the numerical evaluation of Bessel functions in the case of general real order and argument have been implemented on the department's VAX 11/750. These routines utilize power series, phase-amplitude, recursive, and large-order asymptotic expressions, respectively, to approximate Bessel functions of both the first and second kinds ($J_\nu(z)$ and $Y_\nu(z)$), and their derivatives, throughout the z-plane. The routines improve upon and substantially extend other available programs and will be of particular value in several current departmental research efforts.

John Collings

Graduate Student

I am working with Dave Wollkind analyzing dynamical systems in the life sciences. We are using nonlinear stability techniques to study predator-prey ecosystems. We are making extensive use of the VAX for plotting (using PLOT79 and the Talaris, Tektronix, and Lexidata terminals) and solving differential equations (using SIMPLE and the RKF45 software).

Ken Davis

Graduate Student

Under Professor Kallaher's instruction I am looking for a Cartesian Group (C,T,0) of order 27. (See below). It is an open question if one exists. We are using the VAX to generate possible multiplication tables and check if they satisfy the requirements.

A Cartesian group (C,T,0) is such that

i) (C,T) is a group with identity 0

ii) $(C - \{0\}, \cdot)$ is a loop with identity 1 such that

$0 \cdot a = a \cdot 0 = 0 \ \sqrt{a}$

$- ax + bx = c$ has ! solution for all a, b, c $\quad a \neq b$

$- xa + xb = c$ has ! solution for all a, b, c $\quad a \neq b$

and in particular

Both right and left distributive laws fail.

Dave Elwood

Graduate Student

I have used the VAX for:

1) Doing assignments for classes,

2) Research for my Master's project in which it was used for solving the heat equation in 2 space dimensions. It would sometimes take 24 hours of CPU time to run the program. We also used the Soltec plotter and Tektronix to plot the data.

3) For my R.A. for Dr. Mifflin. This research is a program for optimization, in particular, minimization of a function of n-variables.

Danny Lau

Graduate Student

Currently, we are developing some conic algorithms in solving unconstrained dense minimization problems. I'll be using the VAX extensively in the summer for testing various strategies with a standard set of test problems.

By the end of summer, we will be looking at large-scale problems. Hopefully, we'll be getting some good theoretic results in our research to the sparse cases. Then we'll again develop different strategies in solving sparse (large-scale) minimization problems.

In other words, I'll be using the VAX throughout the summer and also the next school year.

Mike Kallaher

Faculty

The VAX is being used to search for projective planes of order 27 which are coordinatized by proper Cartesian systems. It also is being used to look for such planes of a prime order p with p = 11, 13, 17, or 19. This is long term search. The project involves Ken Davis.

Julie Lutz

Faculty

The VAX was used to catalogue and analyze observational data on the symbiotic star AG Draconis. Symbiotic stars are binary stars that exhibit spectra which are a combination of emission lines and the absorption lines characteristic of a cool star. They are particularly difficult to study because their characteristic periods of variation are 100's of days and they exhibit a variety of phenomena in their spectra during the cycles of variation. The data on AG Draconis came from both the optical and the ultraviolet regions of the electromagnetic spectrum. High resolution spectra taken with the 4 m telescope at Kitt Peak National Observatory were analyzed to make identifications of over 700 absorption lines in the spectrum of AG Draconis. The line identifications were used to spectral classify the star and to search for gross peculiarities in the abundances of chemical elements (none were found). The ultraviolet spectra were obtained with the International Ultraviolet Explorer satellite between 1979 and 1985. The spectra were used to determine physical conditions in the emission line region (e.g. temperatures, densities) and to put some constraints on the characteristics of the binary system. This work was done in collaboration with Dr. Thomas Lutz and

Julie Lutz
(continued)

graduate students Mr. James Dull and Mr. David Kolb.

The VAX was used to process spectral data obtained with the 1.5 m and 0.9 m telescopes at Cerro Tololo Interamerican Observatory from 1983 to 1985. The purpose of the observing program was to find distances to and the spectral characteristics of a group of 12 objects that had been classified as both symbiotic stars and planetary nebulae. Usually planetary nebulae have extended shells of material, but if the expansion of the shell is just starting, they can look like a stellar object with a spectrum similar to that of a symbiotic star. The goal of this program was to distinguish between these two categories of objects. The data obtained over two observing seasons were analyzed with the VAX and corrected for the effects of interstellar reddening. For objects which turned out to be young planetary nebulae, some physical properties were derived. This work was done in collaboration with Mr. James Dull and is part of his M.S. thesis which is due to be finished in July 1986.

The VAX was used by Mr. David Kolb who obtained his M.S. degree in May 1986. Mr. Kolb did his thesis on finding distances to planetary nebulae and he used the VAX for the reduction and analysis of observations obtained at Manastash Ridge Observatory and Mt. Laguna Observatory.

Thomas E. Lutz

Faculty

The following list is not in any particular order.

An examination of the calibration of the Period-Luminosity Law for Cepheid variable stars. The data available appeared to result in a unstable solution, and this was confirmed using the MATHVAX.

Ongoing project: telescopes must be "guided" in order to keep pointing at a particular object. The tracking drives which are built are not quite accurate enough. The way to do this is to keep the telescope pointed at a nearby star, and if the star drifts away from a reference position (fiducial mark; cross hair) then move the telescope back. To do this with a computer means developing an algorithm for detecting the center of a star. This is equivalent to finding the center of a "noisy" bi-variate normal distribution which is presented in digitized form. It is a problem in that the noise may be comparable to the data in each bin. But the biggest problem is that it must be done in real time. Thus speed is of importance as well as accuracy. Results to date show that median and mode-like estimators are both faster and more robust than the mean, with noisy data. This project is related to WSU's participation in the ARC telescope project.

The ability to use KERMIT to contact BITNET, and our ability to dial into other VAXes has helped tremendously in generating a proposal with astronomers at Yale, University of North Carolina and Lick Observatory for use of the Hubble Space Telescope.

We have acquired magnetic tape copies of the Yale Bright Star Catalogue, and the New Yale Catalogue of Trigonometric Parralaxes. These data sets have been downloaded to the disks, where they are ready for instant use by

Thomas E. Lutz
(continued)

the faculty and graduate students. They are being used to assemble data sets for calibrating the luminosities of G- and K-giant stars.

A large modeling program has been written for testing different statistical schemes which have been proposed for stellar luminosity calibration (1 Mbyte). This program is being used to study four different methods under different conditions (number of stars, differences in limiting magnitudes and differences in the spread in the true luminosities of the stars).

Allan H. Marcus
Faculty

We have extensively used this system with the SAAM/CONSAM programs to model the effects of lead on indicators of hematotoxicity in humans. The research was supported by the National Institute of Environmental Health Sciences under grant ES03473, "Toxicokinetic Models for Lead in Human Populations". The SAAM (batch) and CONSAM (interactive) programs estimate parameters for systems of coupled nonlinear ordinary differential equations for which linear combinations of state variables are measured (with error) at irregular time points. Available data sets on human subjects exposed to lead provide concentrations of blood lead, erythrocyte protoporphyrin, and delta-aminolevulinic acid dehydratase activity. These data sets are extremely large with respect to the number of data points, kinetically distinct compartments, and unknown parameters. The nonlinear effects include saturable kinetics and time delays. The systems are very stiff. Thus, each iteration requires from 3 minutes to several hours of CPU time. In order to better

Allan H. Marcus
(continued)

study parameter estimation uncertainty, we have extended SAAM to include two new data-dependent methods of uncertainty estimation, the *jackknife* estimation method and the *bootstrap* estimation method (see B. Efron, *The Jackknife, the Bootstrap, and Other Resampling Plans*, SIAM Monograph No. 38, 1982). This appears to be the first installation of these new statistical methods in any routine compartmental analysis program. The programmer was my research assistant, Mr. Mark Quigley. We are preparing three research publications based on these results.

Robert Mifflin
Faculty

During 1985-86 we have utilized the Mathematics Department VAX Computer in our research effort to develop efficient computational methods for solving constrained optimization problems defined with problem functions that are not everywhere differentiable.

We have been working on descent methods that combine polyhedral and quadratic approximation in such a way as to guarantee convergence for very general functions and to obtain rapid convergence for important special cases of nonsmooth functions.

Excellent progress has been made on the single variable case. For example, we solved a real-world resource allocation problem having five bounded decision variables and one linear constraint with less total effort than that required by several good nonlinear programming codes. To accomplish this we used a dual (or nested decomposition) approach which led to a single variable outer problem and a five variable inner problem that separates into five independent single variable problems with upper and lower bounds. We successfully used our FORTRAN subroutine PQ1 to solve both the outer problem and the corresponding sequence of inner subproblems. Actually, we solved two versions of this problem, one with the original smooth objective and the other with a piecewise affine approximation to the objective, in order to show the effectiveness of our algorithm on both types of problems.

We are currently working on generalizations of the single variable algorithm to the n-variable case. Some of the recent numerical work on multivariable test problems has led us to revise our algorithm ideas and come closer to the realization of a better than linearly convergent method.

Mike Moody
Faculty

My use of the VAX:

1. To solve multidimensional systems of nonlinear difference equations which arise in connection with modeling gene-frequency evolution in geographically structured populations.

2. To solve a nonlinear PDE which also arises in the above context.

3. To solve constrained optimization problems for optimal egg-laying strategies in an ecological model.

4. The analysis of DNA sequence patterns.

Tyre A. Newton
Faculty

I have used the VAX on the following two problems:

1. This is a detailed study of an apparent double strange attractor found in a system from classical physics, reported in, R. B. Leipnik and T. A. Newton, Double strange attractors in rigid body motion with linear feedback control, Phys. Lett. 86A (1981) 63-66. Poincare plots are made in the distance-feedback gain parameter plane. This distance is that from the Poincare point to the vertical axis in the Poincare plane which contains the z-axis and the center restpoints about which the upper of the two attractors circulate. Such plots reveal how the orbits vary with the feedback gain parameter. These plots have been made with two digital computers (the VAX and the Dept. of Mech. Eng.'s PRIME) using different integration packages, backed up with analog computer plots for comparison. Several power spectra have been generated for the more spectacular cases where chaos degenerates into limit cycles. It now appears that the u-per attractor itself consists of two attractors that overlay one another, their union forming a "stalk". For some values of the parameter, chaos has degenerated into a single limit

Tyre A. Newton
(continued)

cycle common to both attractors, while for other parameter values the chaos degenerates into two distinct limit cycles. Dan Martin of the WSU Mechanical Engineering Department and I will present our results at the Engineering Foundation conference, QUALITATIVE METHODS FOR THE ANALYSIS OF NONLINEAR DYNAMICS, June 8-13, 1986, at New England College, Henniker, New Hampshire.

2. We are using the VAX to study fractal geometry as exhibited by iterations in the complex plane and the resulting Mandelbrot sets. Slides made from some of these plots have been shown in a paper presented to the, WASHINGTON COMMUNITY COLLEGE MATHEMATICS CONFERENCE, April 25-26, 1986, Alderbrook Resort, WA.

Edward Pate
Faculty

Reaction-diffusion models in developmental biology.

These studies investigated model systems involving both the diffusion and kinetic interaction of chemical species, and the applicability of these systems to phenomena in developmental biology. One biological system considered was the cellular slime molds. Previously analyzed reaction-diffusion, partial differential equations were extended to include terms accounting for both chemotactic, cellular migration and the secretion of an inhibitor of chemotaxis in the slug stage of development. Numerical simulation of the solutions indicates that the model is able to account very well for experimentally observed phenomena, suggesting that even with the increased complexity introduced by chemotactic cell sorting, positional-information-dependent differentiation remains an important aspect of developmental pattern formation. Additionally, the chemoattractant environment in the vicinity of aggregating slime mold cells was investigated. Partial

Edward Pate
(continued)

differential equations were developed to describe the secretion of a pulse of chemoattractant by one cell, the diffusion of the chemoattractant in the extracellular space along with attractant degradation at the surface of a neighboring receiving cell. Numerical techniques were used to analyze the equations. Simulations with experimentally determined kinetic and physical parameters indicate the surpressing result that the receiving cell sits at a local minimum of chemoattractant concentration due to degradation at its surface. The implication on chemotaxis of this unexpected finding are presently being investigated.

An additional developmental system which has been considered is the amphibian embryo. We have shown through numerical simulation that reaction-diffusion equations accurately describe the formation of size-regulated pattern elements in the dorsal-to-ventral sequence in amphibian gastrula. This is true in both normal and symmetric, bipolar embryos generated by implantation of a dorsal lip zone into a ventral zone of a late blastula stage embryo. Experimentally it has been observed that transplantation of developed somite tissue into pregastrula embryos results in fully developed embryos having less somite and an excess of lateral plate tissue when compared with normal embryos. Simple reaction-diffusion models generating only positional information cannot explain this observation. Reaction-diffusion systems have been developed which instead model development as a wave of tissue specification propagating in a dorsal-to-ventral fashion. Numerical analysis of the equations indicates that this type of model duplicates the proportioning evident in polar and bipolar embryos and also, due to cell-type feedback necessary for wave propagation, also results in a

Edward Pate
(continued)

change in cellular proportions in the presence of additional, grafted somite . These models are presently being extended in an attempt to explain development in the more complex situation observed in the marine organism, Hydra.

Tom Richmond
Graduate Student

I am using $\mathrm{LAT_EX}$ on the VAX 11-750 and the Talaris laser printer to process about 100 pages of text describing my original research.

Shingmin Wang
Graduate Student

I feel strongly that the VAX in our department is very helpful to my research on the *Liquid Phase Electro-Epitaxial Growth of Dilute Binary Substance* (for my dissertation under Dr. D. Wollkind). A considerable amount of numerical experiments has been added to the traditional theoretical analysis, which have verified some conjectures and revealed some new features challenging our understanding. The intercommunication has in fact inspired our research.

I would also like to point out that I have implemented quite a few other numerical algorithms on the VAX system. Without doing this one could hardly get the right feeling of what he studied in his class.

It would be unfair not to mention the capability the VAX has to produce high quality prints (such as this) and pictures. It has also contributed very much to my work.

So, in general, I would say that the VAX is of essential importance to our applied mathematicians in both teaching and research activities.

Steve Rummel

Graduate Student

Use of VAX: Classes: Math 544, 448, 512.

Research for Dr. Wollkind. Plots of systems of differential equations for predator prey model.

David Wollkind

Faculty

During the past two years I have used the VAX for scientific computing related to the investigation of two different problems in mathematical modeling. The first deals with interfacial pattern generation during plane-front solidification of a dilute binary substance under the influence of an imposed temperature gradient and electric current. This is a model for the industrially important process of liquid phase electro-epitaxial growth of thin layers of semi-conductor material. My second area of interest is the modeling of exploitation ecosystems for the purpose of planning biological control strategies. My current research in this area deals with the pattern generational aspects of a diffusive instability for a particular reaction-diffusion system appropriate for modeling a temperature-dependent predator-prey mite agro-ecosystem in fruit trees. This ongoing research program is being conducted with the help of a number of my doctoral students.

Kemble Yates
Graduate Student

I use the VAX chiefly in 3 ways: to numerically solve systems of nonlinear parabolic partial differential equations, plot output, and word-process reports on research results. My research interest is in mathematical modelling--primarily of biological systems. In particular, I have been investigating mathematical models of cell differentiation/pattern formation. The biological observation is that a group of homogeneous, "undifferentiated" cells will consistently differentiate to one of a variety of cell types based on their position in the group. My models assume this "positional information" is provided by nonconstant distributions of chemical "morphogens" in the cells. Mathematically, this is expressed as a system of m th partial differential equations, where m is the m-1 cell types plus one equation for the "undiff cells", and n is the number of chemicals. The equations represent balance laws where, specifically, the chemicals are governed by reaction-diffusion equations:

$$\frac{\partial C_i}{\partial t} = \left[\sum_{i=1}^{m-1} \frac{\partial C_i}{\partial t} \right]$$

$$\frac{\partial C_i}{\partial t} = r_i(\underline{c}, \underline{M}) , \quad i = 1, \ldots, m-1$$

$$\frac{\partial M_i}{\partial t} = D_i \frac{\partial^2 M_i}{\partial x^2} + R_i(\underline{c}, \underline{M}), \quad i = 1, \ldots, n.$$

With appropriate terms, boundary conditions, and initial conditions, these equations form a well-posed (though typically stiff) system. I numerically integrate them by discretizing in space and thus creating a large, banded system of ordinary differential equations, which can be solved via Gear's Method. I use our laser printer, Soltec plotter, and Lexidata terminals along with our plotting software to examine my data. And I use our word processing software with our laser printer to create journal-quality prints of my reports.

III. Future Research Work

This will consist of extensions of the work presented in the previous section.

END

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